Market-making with Search and Information Frictions

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    - faster/easier for traders to contact dealers, see bid/ask quotes
    - technology: migration from voice-based to electronic trading in corporate bonds ▸ more
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- **Question: How will these changes affect market liquidity?**
  - A common metric for liquidity: **bid-ask spread**
  - Can also look at: price impact, volume, ...

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- Two canonical sources of illiquidity
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- Challenge:
  - existing literature studies two frictions **in isolation**
  - need a **unified framework**

This paper

- Develop a unified framework of a dynamic asset market with:
  1. trading frictions
  2. asymmetric information

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- **Focus: reducing trading frictions can lead to wider bid-ask spreads**
  - Static effect: trading frictions $\downarrow \Rightarrow$ competition $\uparrow \Rightarrow$ spreads $\downarrow$ (DGP)
  - Dynamic effect: **trading frictions $\downarrow \Rightarrow$ learning slows** $\Rightarrow$ spreads eventually $\uparrow$ (GM)

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- **Additional contributions:**
  - Tradeoffs shed light on empirical findings on effects of Δ trading frictions
    - e.g., Hendershott and Moulton (2011)
  - Anticipating impact of regulations that reduce info or trading frictions

Literature

Market-making with asymmetric information

- "Small" informed traders, dealers learn from individual trades: Glosten-Milgrom(1985), ...
- "Large" informed trader, dealers learn from aggregate trade: Kyle(1985),...
- This paper: "small" informed traders, dealers learn from aggregate trade, search & market power

Market-making with search frictions

- Full info: Duffie, Garleanu & Pedersen(2005), Lagos & Rocheteau(2009)...
- Private info, private values: Spulber(1996), Lester, Rocheteau & Weill (2015)...
- This paper: private information about common values (adverse selection), learning

Decentralized trading with adverse selection

- Idiosyncratic: Inderst(2005), Guerrieri-Shimer-Wright(2010), Camargo & Lester(2014), Lauermann & Wolinsky(2016), Kim (2017)...
- Aggregate: Wolinsky(1990), Blouin & Serrano(2001), Duffie, Malamud & Manso(2009), Golosov, Lorenzoni & Tsyvinski(2014)...
- This paper: Learning from market-wide activity, effect of info frictions on bid-ask spread

the economic environment

Agents and Assets

- Discrete time, infinite horizon
- A market for a **single** asset, quality (state of the world) is either *l* or *h*
- A continuum of **traders**
  - can hold $q \in \{0, 1\}$ units of the asset
  - with probability $1 - \delta$ in each period, asset matures (game ends)
  - traders have private info about asset quality + their own preferences
- A continuum of **dealers**
  - can hold unrestricted positions (long or short)
  - less informed (ex ante) about asset quality, but learn from trading activity

Preferences

Given state of world $j \in \{l, h\}$,

- **trader** i who owns an asset receives:
  - flow payoff $\omega_t + \varepsilon_{i,t}$ per period
  - terminal payoff c_j upon maturity, with $c_h > c_l$

  with

  - $\omega_t \sim F(\omega) =$ market-wide liquidity shock, mean zero, iid over time
  - $\varepsilon_{i,t} \sim G(\varepsilon) =$ idiosyncratic liquidity shock, mean zero, iid over time

- For each unit he holds, **dealer** receives:
  - payoff v_j at maturity, with $v_h > v_l$
  - no liquidity shocks

Search, Prices, and Trade

Each period, trader meets stochastic number $n \in \{0, 1, 2, ...\}$ of dealers

Prob(meet $n \geq 1$ dealer) $= \pi$

Conditional on meeting at least one dealer,

- Prob(meet $n = 1$ dealer) $= \alpha_m$ ("monopolist meeting")
- Prob(meet $n \geq 2$ dealer) $= \alpha_c = 1 - \alpha_m$ ("competitive meeting")

Dealers observe number of competing dealers but not asset quality/trader preferences

- offer to buy at bid price B_t^k, sell at ask price A_t^k for $k \in \{c, m\}$
- trader accepts or rejects.
- if she rejects, no trade occurs in that period.

Information and Learning

After trades occur in each period, dealers observe total trading volume

Two sources of uncertainty for dealers:

1. asset quality: common value
2. aggregate liquidity shock: private value

$\Rightarrow$ volume is a noisy signal about asset quality

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Dealers are informationally small and all have common beliefs

- Beliefs summarized by $\mu_t \equiv \text{Prob}_t(j = h)$

optimal behavior and equilibrium

Traders' Optimal Behavior

- $W_{j,t}^{q} \equiv$ value of owning $q \in \{0, 1\}$ units of quality $j \in \{l, h\}$ asset at t

- Given bid and ask prices (B_t^k, A_t^k), $k \in \{m, c\}$, and shocks $(\varepsilon_{i,t}, \omega_t)$,
  - Owner should sell if $\varepsilon_{i,t}$ sufficiently small, hold otherwise:

$$B_t^k + W_{j,t+1}^0 \geq \varepsilon_{i,t} + \omega_t + W_{j,t+1}^1$$

  - Non-owner should buy if if $\varepsilon_{i,t}$ sufficiently large, do nothing otherwise:

$$-A_t^k + \varepsilon_{i,t} + \omega_t + W_{j,t+1}^1 \geq W_{j,t+1}^0$$

Traders' Optimal Behavior

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  - Owner should sell if $\varepsilon_{i,t}$ sufficiently small, hold otherwise:
  $$B^k_t + W^0_{j,t+1} \geq \varepsilon_{i,t} + \omega_t + W^1_{j,t+1}$$
  - Non-owner should buy if if $\varepsilon_{i,t}$ sufficiently large, do nothing otherwise:
  $$-A^k_t + \varepsilon_{i,t} + \omega_t + W^1_{j,t+1} \geq W^0_{j,t+1}$$
- $R_{j,t} = W^1_{j,t} - W^0_{j,t} \equiv$ reservation value at t when quality is $j \in \{l,h\}$

Traders' Optimal Behavior

- Owner i sells in a $k \in \{m, c\}$ meeting iff

$$\epsilon_{i,t} \leq \underline{\epsilon}_{j,t}^{k} \equiv B_t^k - R_{j,t+1} - \omega_t$$

- Non-owner i buys in a k meeting iff

$$\epsilon_{i,t} \geq \bar{\epsilon}_{j,t}^{k} \equiv A_t^k - R_{j,t+1} - \omega_t$$

- Reservation values satisfy

$$R_{j,t} = (1-\delta)c_j + \delta \mathbb{E}\left[R_{j,t+1}\right] + \delta\pi\mathbb{E}\left[\underbrace{\Omega_{j,t+1}}_{\text{Net option value}}\right]$$

where

$$\Omega_{j,t} = \sum_{k=c,m} \alpha^k \left[\underbrace{\max\{B_t^k - R_{j,t+1} - \omega_t - \varepsilon_{i,t}, 0\}}_{\text{option to sell}} - \underbrace{\max\{-A_t^k + R_{j,t+1} + \omega_t + \varepsilon_{i,t}, 0\}}_{\text{option to buy}}\right]$$

Aggregate Positions

$N^q_{j,t}$ = measure of traders holding $q \in \{0, 1\}$ units of asset when quality is $j \in \{l, h\}$

$$N^1_{j,t+1} = \left\{ N^1_t \left[\underbrace{1-\pi}_{\text{no meeting}} + \underbrace{\pi \left(1 - \sum_{k=c,m} \alpha^k G(\underline{\varepsilon}^k_{j,t})\right)}_{\text{meeting, no sell}} \right] + N^0_t \underbrace{\pi \left(1 - \sum_{k=c,m} \alpha^k G(\overline{\varepsilon}^k_{j,t})\right)}_{\text{meet \& buy}} \right\}$$

$$N^0_{j,t+1} = \left\{ N^1_t \pi \sum_{k=c,m} \alpha^k G(\underline{\varepsilon}^k_{j,t}) + N^0_t \left[1 - \pi + \pi \sum_{k=c,m} \alpha^k G(\overline{\varepsilon}^k_{j,t}) \right] \right\}.$$

Dealers observe past volume

$\Rightarrow$ they know N^q_t when setting (B^k_t, A^k_t).

Monopolist Dealer's Prices

Dealer with a captive customer chooses (A_t^m, B_t^m) to maximize

$$\mathbb{E}_{j,\omega}\left[\frac{N_t^0}{N_t^0+N_t^1}\left(1-G(\overline{\varepsilon}_{j,t}^m)\right)(A_t^m-v_j)+\frac{N_t^1}{N_t^0+N_t^1}G(\underline{\varepsilon}_{j,t}^m)(v_j-B_t^m)\right]$$

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Why? we find conditions s.t. no motive for experimentation, no benefit to waiting

- Pricing decision is static
- Sell (buy) choice unaffected by ask (bid) $\Rightarrow$ separates the bid/ask problems
- Aggregate positions known $\Rightarrow$ irrelevant for pricing, only beliefs μ_t matter

▸ No Experimentation

Key assumptions s.t. market-wide info dominates learning from individual meeting

- Both traders and dealers are small, so take future beliefs as given
- Dealers can hold unrestricted positions, have deep pockets
- Support of shocks "large enough"

Monopolist Dealer's Prices (given beliefs μ_t)

As a result, optimal monopoly prices satisfy:

$$A_t^m = \mathbb{E}_j v_j + \underbrace{\frac{1 - \mathbb{E}_{j,\omega}\left[G\left(\overline{\varepsilon}_{j,t}^m\right)\right]}{\mathbb{E}_{j,\omega}\left[g\left(\overline{\varepsilon}_{j,t}^m\right)\right]}}_{\text{market power}} + \underbrace{\mu_t(1-\mu_t)(v_h - v_l)\frac{\mathbb{E}_\omega\left[g\left(\overline{\varepsilon}_{h,t}^m\right) - g\left(\overline{\varepsilon}_{l,t}^m\right)\right]}{\mathbb{E}_{j,\omega}\left[g\left(\overline{\varepsilon}_{j,t}^m\right)\right]}}_{\text{asymmetric information}}$$

$$B_t^m = \mathbb{E} v_j - \frac{\mathbb{E}_{j,\omega}\left[G\left(\underline{\varepsilon}_{j,t}^m\right)\right]}{\mathbb{E}_{j,\omega}\left[g\left(\underline{\varepsilon}_{j,t}^m\right)\right]} - \mu_t(1-\mu_t)(v_h - v_l)\frac{\mathbb{E}_\omega\left[g\left(\underline{\varepsilon}_{l,t}^m\right) - g\left(\underline{\varepsilon}_{h,t}^m\right)\right]}{\mathbb{E}_{j,\omega}\left[g\left(\underline{\varepsilon}_{j,t}^m\right)\right])}.$$

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Competitive Prices

Bertrand competition ⇒ zero profits (*a la* Glosten-Milgrom)

$$A_t^c = \frac{\mathbb{E}_{j,\omega}\left[v_j\left(1 - G(\bar{\varepsilon}_{j,t}^c)\right)\right]}{\mathbb{E}_{j,\omega}\left[\left(1 - G(\bar{\varepsilon}_{j,t}^c)\right)\right]}$$

$$B_t^c = \frac{\mathbb{E}_{j,\omega}\left[v_j G(\underline{\varepsilon}_{j,t}^c)\right]}{\mathbb{E}_{j,\omega}\left[G(\underline{\varepsilon}_{j,t}^c)\right]}$$

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Monopoly vs. Competitive (Ask) Prices

$$A_t^m = \mathbb{E}_j v_j + \underbrace{\frac{1 - \mathbb{E}_{j,\omega}\left[G\left(\overline{\varepsilon}_{j,t}^m\right)\right]}{\mathbb{E}_{j,\omega}\left[g\left(\overline{\varepsilon}_{j,t}^m\right)\right]}}_{\text{market power}} + \underbrace{Cov\left(\frac{g\left(\overline{\varepsilon}_{j,t}^m\right)}{\mathbb{E}_{j,\omega}\left[g\left(\overline{\varepsilon}_{j,t}^m\right)\right]}, v_j\right)}_{\text{asymmetric information}}$$

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Two key differences:

1. Competitive price has no markup/market power term.
2. PDF vs. CDF:
   - Monopolist's optimal price depends on mass of *marginal* investors
   - Competitive price requires equal profits *on average*

Evolution of Beliefs

Information: Dealers see volume at end of t (buys and sells), or equivalently

$$\underline{\epsilon}_t^k = B_t^k - R_{t+1} - \omega_t \qquad \text{or} \qquad \bar{\epsilon}_t^k = A_t^k - R_{t+1} - \omega_t$$

where $R_{t+1} = R_{j,t+1}$ if asset is of quality j

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$$\omega^{\star}_{\iota,t} = S_t - R_{\iota,t+1}$$

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$$\omega_{\iota,t}^{\star} = S_t - R_{\iota,t+1}$$

Beliefs then evolve according to

$$\mu_{t+1} = \frac{\mu_t f\left(\omega_{h,t}^{\star}\right)}{\mu_t f\left(\omega_{h,t}^{\star}\right) + (1-\mu_t) f\left(\omega_{l,t}^{\star}\right)} = \frac{\mu_t}{\mu_t + (1-\mu_t)\frac{f\left(\omega_t + R_{j,t+1}(\mu_{t+1}) - R_{l,t+1}(\mu_{t+1})\right)}{f\left(\omega_t + R_{j,t+1}(\mu_{t+1}) - R_{h,t+1}(\mu_{t+1})\right)}}$$

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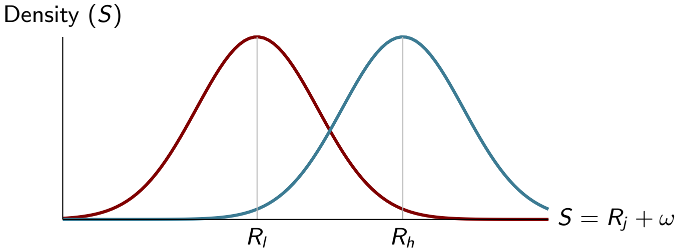
Learning process depends on $R_{h,t+1} - R_{l,t+1}$

- Trading typically more informative when the reservation values are very different

Learning: Picture



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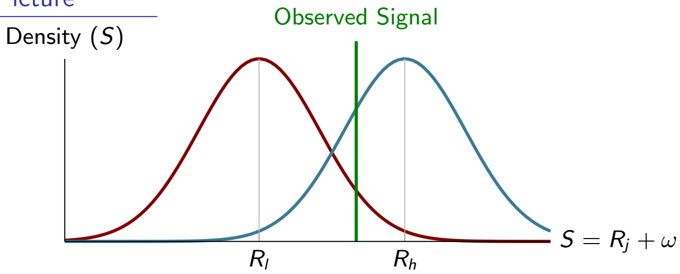

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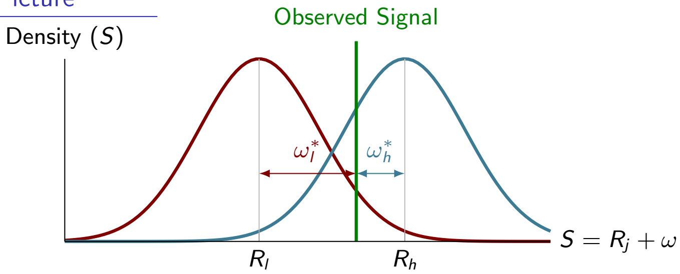

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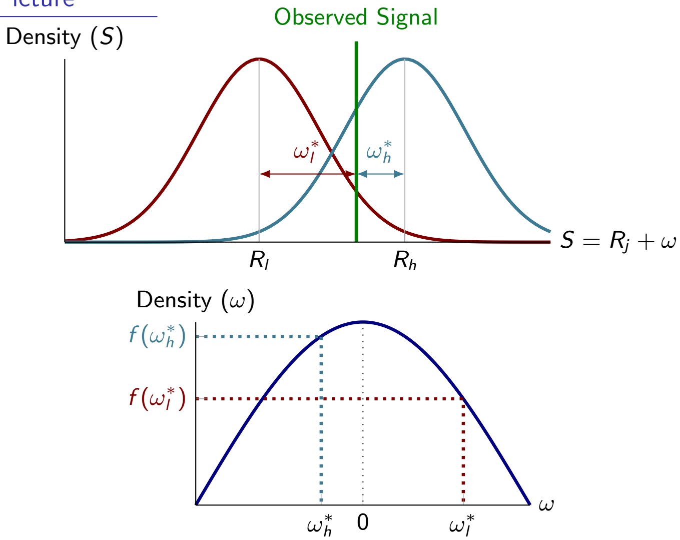

- Belief evolution depends on basic signal extraction
- Easy to see signal extraction problem more difficult if reservation values close together

Equilibrium

A recursive equilibrium is a collection of functions for

1. Reservation values: $R_j(\mu) \quad j \in \{h, l\}$
2. Thresholds: $\underline{\varepsilon}_j^k(\mu, \omega)$, $\overline{\varepsilon}_j^k(\mu, \omega) \quad k \in \{c, m\}$
3. Prices: $A^k(\mu)$, $B^k(\mu)$
4. Beliefs: $\mu'(\mu, \omega)$
5. Demographics: $N_j^0(\mu, \omega)$, $N_j^1(\mu, \omega)$

such that

1. Reservation values are consistent with future beliefs and prices
2. Given beliefs and prices, thresholds are optimal for traders
3. Given beliefs and thresholds, prices are optimal for dealers
4. Beliefs evolve according to Bayes' rule
5. Demographics evolve consistent with prices, thresholds

a tractable case

The Uniform-Uniform Model

Assumptions:

1. $v_j = c_j$ for $j \in \{l, h\}$
2. $\varepsilon_{i,t} \sim U(-e, e)$ and $\omega_t \sim U(-m, m)$
3. e and m are sufficiently large s.t. thresholds are always interior

Together, these assumptions simplify both learning and pricing.

Given simple rules for pricing, updating beliefs and prices, we can...

- characterize (unique) equilibrium
- study relationship between search frictions, learning, and spreads

Learning in the Uniform-Uniform Model

Recall: updating equation depends on

$$\frac{f(\omega_l^\star)}{f(\omega_h^\star)} = \frac{f(S - R_l)}{f(S - R_h)}$$

Guess and verify

$$\mu'(\mu, S) = \begin{cases} 0 & \text{if } S \in \Sigma_l(\mu) \equiv [-m + R_l(0), -m + R_h(\mu)) \\ \mu & \text{if } S \in \Sigma_b(\mu) \equiv [-m + R_h(\mu), m + R_l(\mu)] \\ 1 & \text{if } S \in \Sigma_h(\mu) \equiv (m + R_l(\mu), m + R_h(1)] \,. \end{cases}$$

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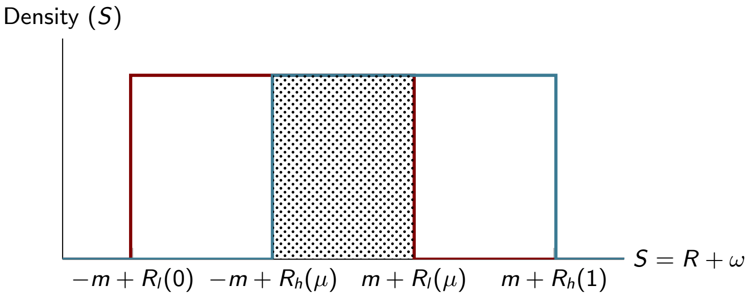
Guess and verify

$$\mu'(\mu, S) = \begin{cases} 0 & \text{if } S \in \Sigma_l(\mu) \equiv [-m + R_l(0), -m + R_h(\mu)) \\ \mu & \text{if } S \in \Sigma_b(\mu) \equiv [-m + R_h(\mu), m + R_l(\mu)] \\ 1 & \text{if } S \in \Sigma_h(\mu) \equiv (m + R_l(\mu), m + R_h(1)] \,. \end{cases}$$



Learning in the Uniform-Uniform Model

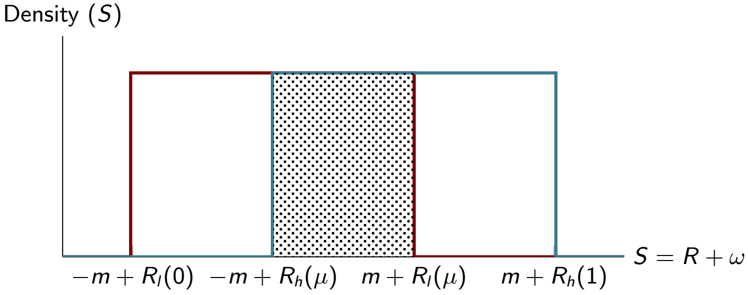
In candidate eqm, learning process summarized by $\mathbb{P}$(quality revealed):

$$p(\mu) = \frac{R_h(\mu) - R_l(\mu)}{2m}.$$

Immediate implication:

Result

Time to learn, $\frac{1}{p(\mu)}$ *increases as* $(R_h - R_l) \downarrow$.



Pricing & Equilibrium in the Uniform-Uniform Model

Given simple learning process and linear demand/supply, prices easy to characterize

Implied bid-ask spread σ given current beliefs $\mu \in (0,1)$:

$$\sigma(\mu) = e - \alpha_c \sqrt{e^2 - 4Cov\left(r_j, v_j\right)}$$

where

$$r_j = p(\mu)R_j\left(\mathbf{1}_{j=h}\right) + (1 - p(\mu))R_j(\mu).$$

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Simple expression allows us to derive properties of spreads

Result

Spread is $\bigcap$-shaped in μ, maximized at $\mu = 1/2$.

Result

Holding μ fixed, spread is decreasing in π.

Reservation Values and Search Frictions

How does a higher π affect spreads?

Crucial channel: effect of π on $R_h - R_l$:

$$R_h - R_l = (1-\delta)\,(c_h - c_l) + \delta\,\mathbb{E}[R'_h - R'_l] + \delta\pi\,\mathbb{E}(\Omega'_h - \Omega'_l)$$

where Ω_j = option value of selling − option value of buying

Result

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Result

$R_h - R_l$ is decreasing in π.

- $\Omega'_h - \Omega'_l < 0$: Option to sell (buy) is worth less (more) when quality is high
- Higher π increases the weight of the net option value, bringing R_h and R_l closer
- Intuition: investors behave more alike in two states when more opportunities to trade
- $\Rightarrow$ less adverse selection (given μ), but also *slower learning*

Result (Putting it all together)

1. *Holding* $\mu \in (0, 1)$ *fixed, spread* $\downarrow$ *as* $\pi \uparrow$ *(Static)*

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2. *Spread big when uncertainty high (*$\mu \approx 1/2$*)*
3. $(R_h - R_l) \downarrow$ *as* $\pi \uparrow$
4. *Learning occurs slower when* $R_h - R_l$ *is small (Dynamic)*

Search Frictions and Spreads

Result (Putting it all together)

1. *Holding $\mu \in (0,1)$ fixed, spread $\downarrow$ as $\pi \uparrow$ (Static)*
2. *Spread big when uncertainty high ($\mu \approx 1/2$)*
3. $(R_h - R_l) \downarrow$ *as* $\pi \uparrow$
4. *Learning occurs slower when $R_h - R_l$ is small (Dynamic)*

Therefore, two opposing effects on spread from decreasing search frictions ($\pi \uparrow$):

- **Static:** spread $\downarrow$ as competition $\uparrow$
- **Dynamic:** $(R_h - R_l) \downarrow \Rightarrow$ learning slows $\Rightarrow$ more uncertainty $\Rightarrow$ spread $\uparrow$

Search Frictions and Spreads

Numerical simulation: $j = h$, $\mu = 1/2$, $\pi \in \{0.25, .75\}$.

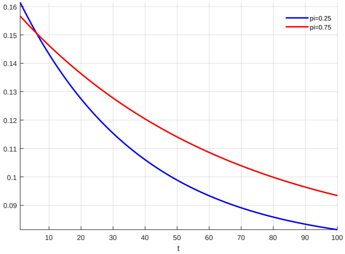


Figure: Average Spread Over Time

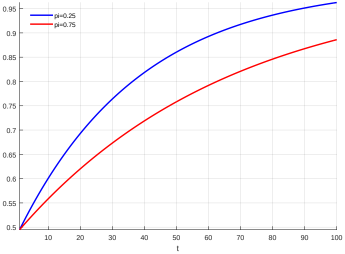


Figure: Average Beliefs Over Time

- $\pi \uparrow$ causes fall in spread in **current period**
- $\pi \uparrow$ causes slower learning, higher spreads in **future periods** spreads

Numerical Example

Generalized Version of Model

Relax previous assumptions on distributions, valuations:

- $\omega_t \sim N(0, \sigma_\omega^2) \qquad \varepsilon_{i,t} \sim N(0, \sigma_\varepsilon^2)$
- $v_j = c_j + \xi$
- Additional, higher order terms complicate analysis

But, model easily solved computationally

- Guess $R_j(\mu)$ for $j = l, h$
- Given R_j, determine dealers' evolution of beliefs μ^+
- Given future beliefs and R_j, compute $A(\mu)$ and $B(\mu)$
- Update guess of R_j until convergence

Parameterization

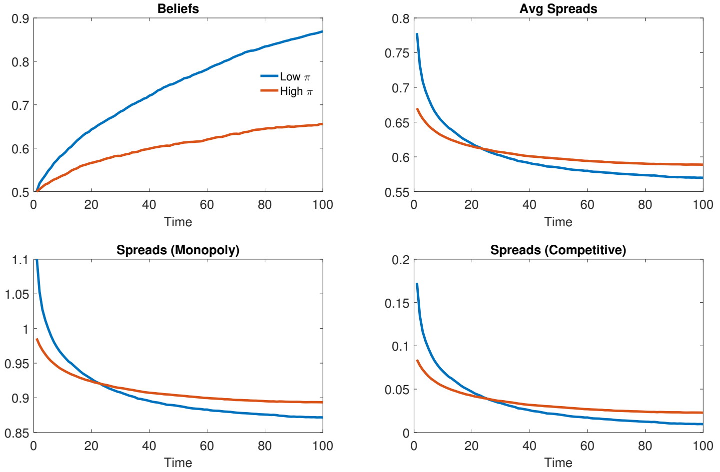
- Parameters approximate evidence from AAA-rated 5-year corporate bond evidence
- No gains to trade (on average) between dealers and traders ($\xi = 0$)
- Model period set to 1 week

Parameter	Value	Target	Source
$v_h - v_l$	\$0.95	Impact of Downgrade	Feldhutter (2012b)
μ_0	0.5	Probability of (AAA $\rightarrow$ AA) Downgrade	S&P
$\sigma_\omega^2 = \sigma_\epsilon^2$	0.16	Avg. Gains to Trade	Feldhutter (2012a)
π	0.55	Match Rates given Poisson	Feldhutter (2012a)
α	0.35		
δ	0.9	sensitivity	

- $\delta = 0.9$ implies trading horizon (conditional on no trade) of 10 weeks

The Normal-Normal Model

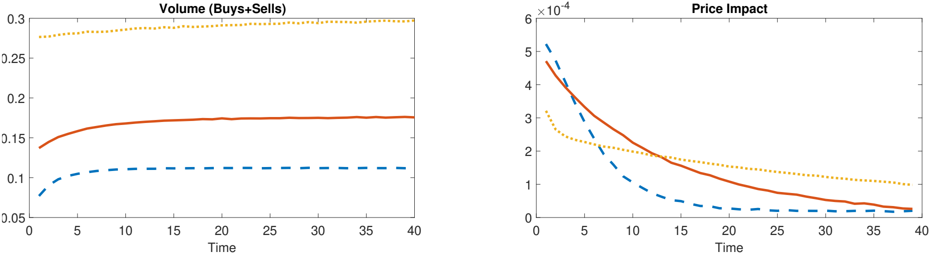
Effect of π (true state is $j = h$)



Higher π $\rightarrow$ Lower $(R_h - R_l)$ $\rightarrow$ Less learning $\rightarrow$ Wider spreads eventually

Other measures of liquidity

Effect of π (true state is $j = h$)



- Price impact behaves similarly to spreads, but not volume
- Note: spreads and volume can move in same direction, as in data

Search vs Info Frictions

Exercise: hit benchmark with shocks to π and $v_l \Rightarrow$ same Δ spread.

Question: are dynamic properties of spread *and* volume informative?

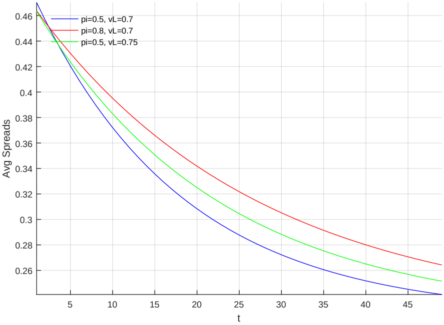


Figure: Spreads

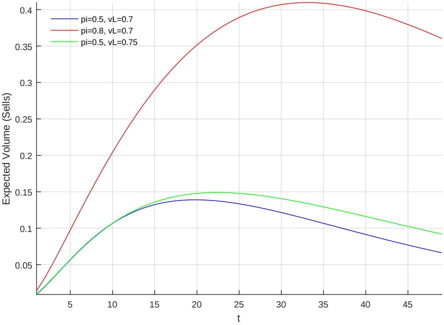
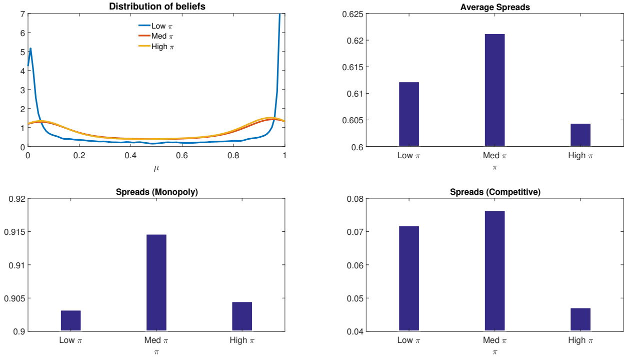


Figure: Volume

Stationary Version

The Normal-Normal model: Stationary Version

- Asset quality j changes over time (with probability $\rho = 0.05\%$)
- Other elements exactly the same as before
- $\Rightarrow$ Non-trivial belief distribution in the long run (stochastic steady state)



low π=0.55, med π= 0.75, high π = 0.95

- Higher π $\rightarrow$ Lower $(R_h - R_l)$ $\rightarrow$ Less learning $\rightarrow$ Wider spreads

Conclusion

A dynamic model with two canonical frictions

- asymmetric information and infrequent trading opportunities/market power

Frictions interact in novel ways

- mitigating one could lead to wider spreads
- model helpful for understanding recent changes in OTC markets

Next steps

- Simulations suggest introduction of TRACE could widen spreads...
- disentangling the two frictions?....

Dealers

- Indexed by $i \in [0,1]$
- They come into each period with $x_{i,t}$ units of the asset
- Payoff:

$$\sum_{s=t}^{\infty}(1-\delta)^{s-t}[-d_{i,t}P_t + q_{i,t}p_t + \delta v_j(x_{i,t} + d_{i,t} + q_{i,t})]$$

  where

$$d_{i,t} \in \{-1, 0, 1\}$$

$$P_t \in \{A_t, B_t\}$$

$$x_{i,t+1} = x_{i,t} + d_{i,t} + q_{i,t}$$

- p_t: price in the interdealer market; competitive

Dealers

- Conjecture that future bid and ask only a function of aggregate information and independent of individual positions.
- Radner: REE in the inter-dealer market $p_t = \mathbb{E}_t \left[v_j | \{d_{i,t}\}_{i \in [0,1]} \right]$.
- Dealers are small:

$$\mathbb{E}_t[v_j | p_t, d_{i,t}] = \mathbb{E}_t \left[v_j | \{d_{i,t}\}_{i \in [0,1]} \right]$$

- Act as if they are short-lived dealers and only care about $\mathbb{E}_t[v_j]$ where expectation is common across all dealers

Back

Experimentation

- From individual trader, dealer can learn at most $R_{j,t} + \omega_t + \epsilon_{i,t}$

- From market volume, dealer will learn $R_{j,t} + \omega$

- Since $\epsilon_{i,t}$ independent of the state, j, information in market volume dominates information that can be learned from a single trade
  - dominates in sense that dealer unwilling to pay any cost to learn $R_{j,t} + \omega_t + \epsilon$

▸ Back

Equilibrium

A recursive equilibrium is a set of functions: $R_j(\mu)$ $A(\mu)$ and $B(\mu)$ s.t.

$$R_j = (1-\delta)\, c_j + \delta\, \mathbb{E}[R_j(\mu_j')] + \delta\, \pi\;\; \Omega_j(\mu)$$

$$A = \frac{\mathbb{E} v_j g(A - R_j(\mu_j') - \omega) + 1 - \mathbb{E} G(A - R_j(\mu_j') - \omega)}{\mathbb{E} g(A - R_j(\mu_j') - \omega)}$$

$$B = \frac{\mathbb{E} v_j g(B - R_j(\mu_j') - \omega) - \mathbb{E} G(B - R_j(\mu_j') - \omega)}{\mathbb{E} g(B - R_j(\mu_j') - \omega)}$$

where

$$\mu_j' = \frac{\mu}{\mu + (1-\mu)\, \mathcal{L}_j(\omega, R_h(\mu_j') - R_l(\mu_j'))}$$

$$\Omega_j(\mu) = \mathbb{E}\left[\max(B(\mu) - R_j(\mu_j') - \omega - \epsilon, 0) - \max(R_j(\mu_j') + \omega + \epsilon - A(\mu), 0)\right]$$

Back

Equilibrium

Start with a guess for $R_j(\mu) \rightarrow$ beliefs

$$\mu'_j = \frac{\mu}{\mu + (1-\mu)\, \mathcal{L}_j(\omega, R_h(\mu'_j) - R_l(\mu'_j))}$$

▸ Back

Equilibrium

Compute optimal prices: $A(\mu)$ and $B(\mu)$

$$A = \frac{\mathbb{E} v_j g(A - R_j(\mu_j') - \omega) + 1 - \mathbb{E} G(A - R_j(\mu_j') - \omega)}{\mathbb{E} g(A - R_j(\mu_j') - \omega)}$$

$$B = \frac{\mathbb{E} v_j g(B - R_j(\mu_j') - \omega) - \mathbb{E} G(B - R_j(\mu_j') - \omega)}{\mathbb{E} g(B - R_j(\mu_j') - \omega)}$$

$$\mu_j' = \frac{\mu}{\mu + (1-\mu)\ \mathcal{L}_j(\omega, R_h(\mu_j') - R_l(\mu_j'))}$$

Back

Equilibrium

Update/verify the guess

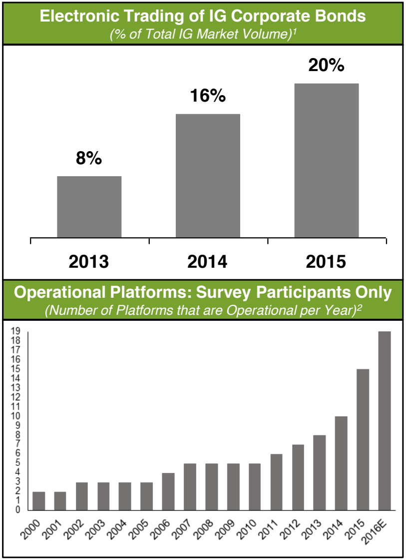
$$
\begin{aligned}
R_j &= (1-\delta)\, c_j + \delta\, \mathbb{E}[R_j(\mu_j')] + \delta\, \pi \;\; \Omega_j(\mu) \\
A &= \frac{\mathbb{E} v_j g(A - R_j(\mu_j') - \omega) + 1 - \mathbb{E} G(A - R_j(\mu_j') - \omega)}{\mathbb{E} g(A - R_j(\mu_j') - \omega)} \\
B &= \frac{\mathbb{E} v_j g(B - R_j(\mu_j') - \omega) - \mathbb{E} G(B - R_j(\mu_j') - \omega)}{\mathbb{E} g(B - R_j(\mu_j') - \omega)}
\end{aligned}
$$

$$
\mu_j' = \frac{\mu}{\mu + (1-\mu)\, \mathcal{L}_j(\omega, R_h(\mu_j') - R_l(\mu_j'))}
$$

$$
\Omega_j(\mu) = \mathbb{E}\left[\max(B(\mu) - R_j(\mu_j') - \omega - \epsilon, 0) \; - \; \max(R_j(\mu_j') + \omega + \epsilon - A(\mu), 0)\right]
$$

Back

Corporate Bond Market (from SIFMA report)



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